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AN INSTRUMENT FOR MEASURING THE THERMAL PENETRATION PROPERTY ρck*

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Abstract—An instrument which measures the product of density, specific heat, and thermal conductivity of solid materials is described, a parameter which frequently occurs in analyses of unsteady heat conduction processes. The results of measurements are reported which demonstrate the accuracy which can be obtained.

NOMENCLATURE

- c, specific heat [J/kg K];
- Fo, Fourier number ($\alpha t/L^2$);
- k, thermal conductivity [W/m K];
- L, thickness of silastic rubber [m];
- r, radial coordinate [m];
- R, dimensionless radial coordinate (r/L);
- t, time;
- T, temperature [K];
- z, axial coordinate [m];
- Z, dimensionless axial coordinate (z/L).

Greek symbols

- α, thermal diffusivity ($k/\rho c$);
- ρ, density [kg/m^3];
- θ, dimensionless temperature $T-T_1^0/T_2^0-T_1^0$.

Subscripts and superscripts

- 1, measured material;
- 2, sampler material;
- 3, insulation;
- m, interface;
- 0, initial value.

INTRODUCTION

TODAY an important consideration in the design of buildings is energy conservation. To reduce the energy required for heating in winter and cooling in summer one has to consider the ventilation load (intake and infiltration air) and heat transmission through the structure envelope. Both heat fluxes can be greatly reduced if the building is constructed underground, as has been pointed out by Bligh [1]. For an accurate prediction of the energy requirements one also has to consider the energy storage within the envelope of the structure; in an underground building this must include the soil surrounding the building [2]. The heat flux into and out of the wall of a building caused by a timewise temperature variation at the surface depends on the product of density ρ, specific heat per unit mass c and thermal conductivity k, a property which will be referred to as the "thermal penetration property" following the suggestion of Krischer and Esdorn [3]. An analysis of the unsteady heat-conduction processes caused by a variation of the surface temperature results, for instance, in an equation of the form

$$q = (\rho ck)^{1/2} f(t) \Delta T \tag{1}$$

in which q is the heat flux per unit time and area, ΔT denotes a characteristic temperature difference and $f(t)$ denotes a function of time. It should be observed that the material property ρck, the thermal penetration property, is involved.

This paper describes an instrument which permits the experimental determination of the thermal penetration property not only for samples of materials but also for *in situ* materials such as building walls, roofs or floors. It also is found that the value of each of these three properties of ρck increases with increasing moisture content within the material. In this way the instrument gives an important indication of the moisture content locally or timewise in a building wall when repeated measurements are made. For example, Fig. 1 shows the variation of the product ρck with moisture content for foam concrete [4].

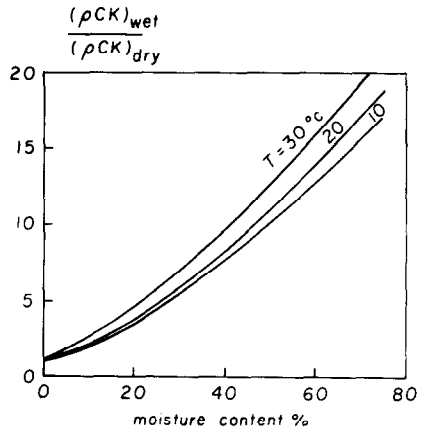

FIG. 1. Variation of the thermal penetration parameter ρck with moisture content for foam concrete. Calculated from independent values of ρ, c and k from Luikov [4].

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Moisture in a material can be detected by nuclear moisture meters in which a small radioactive source releases neutrons and the instrument measures the neutron back-scatter produced by hydrogen atoms within the material. These meters are expensive and require care in handling because they use radioactive materials and are not accurate unless calibrated for each material being measured.

PRINCIPLES OF THE THERMAL PENETRATION PROPERTY SAMPLER

When two semi-infinite solids initially at constant but different temperatures T_1^0 and T_2^0 and with constant thermal properties, are brought into contact, as indicated in Fig. 2, the interfacial temperature T_m, will

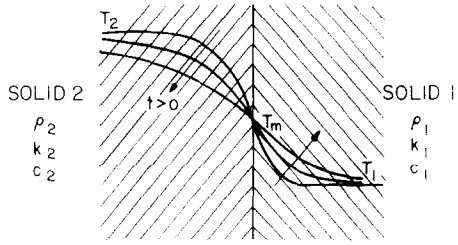

FIG. 2. Temperature profiles in two semi-infinite materials of different temperatures suddenly brought into contact.

immediately assume and thereafter remain at a constant value independent of time as heat flows from the higher temperature solid into the lower temperature solid. T_m is given by

$$(T_m - T_1^0)/(T_2^0 - T_m) = (\rho_2 c_2 k_2/\rho_1 c_1 k_1)^{1/2} \quad (2)$$

in which the superscript 0 indicates the temperatures at the initial time of contact [5].

If the material 2 is selected with known thermal properties $(\rho_2 c_2 k_2)$ and the initial temperatures T_1^0 and T_2^0 as well as the interface temperature T_m are measured, the parameter $(\rho_1 c_1 k_1)$ can be obtained.

The sampler is based on the physical processes discussed above. Figure 3A shows a cross-section of the sampler. A disk manufactured from an appropriately selected material is brought to a desired uniform temperature T_2^0 by placing it on the hot plate shown in Fig. 3B. Heat is provided by a resistance coil in the hot plate; a second coil on top of the sampler is provided to reduce the time required to reach a uniform temperature in the sampler material disk during heating. When the temperature T_2^0 of the material has become uniform, a situation which can be checked by the three thermocouples shown in the figure, the sampler is placed against material 1 for which the thermal penetration property is required and the initial temperature T_1^0 of which has been measured. From the temperatures T_1^0, T_2^0 and T_m and the known property $(\rho ck)_2$ of the sampler material, the thermal penetration property of material 1 can be obtained through equation (2). The sampler material should provide a good contact when it is pressed against material 1. Otherwise, the temperature T_m will change in time. The sampler in Fig. 3 is enclosed in a pinewood block to minimize heat losses from all surfaces except the one in contact with the material 1.

The interface temperature T_m is located half way between the temperatures T_1^0 and T_2^0, when the thermal penetration property of material 1 has the same value as the penetration property of the sampler material. Otherwise, the temperature T_m will be closer to the temperature of whichever material has the higher value of ρck. A situation in which any of the temperature differences $T_m - T_1^0$ or $T_2^0 - T_m$ becomes

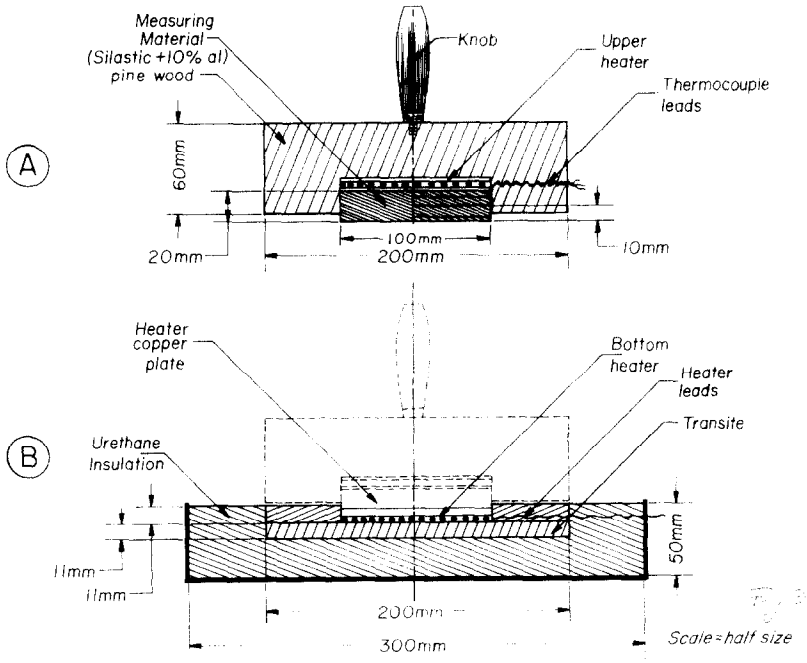

FIG. 3. Cross-section of the thermal penetration property sampler.

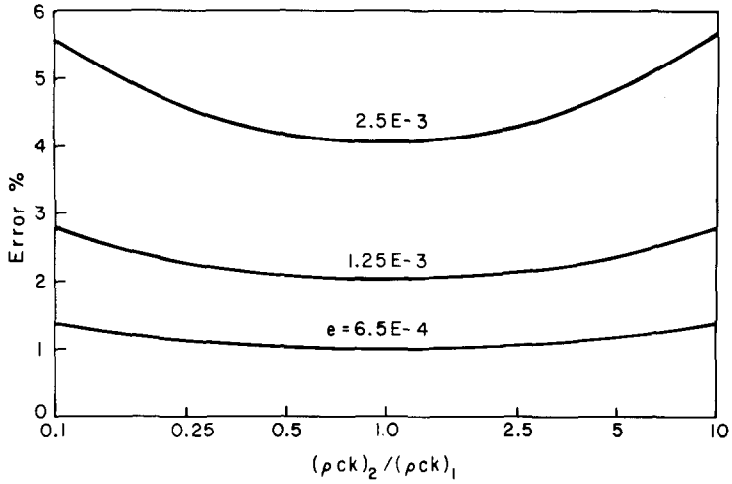

FIG. 4. Percentage error with which the thermal penetration property can be measured.

too small should be avoided since this would reduce the accuracy with which the penetration property can be determined. The ordinate of Fig. 4 shows the error in percent with which the penetration property can be determined within a limited accuracy of the temperature readings. The abscissa of the figure presents the ratio of the penetration properties $(\rho ck)_2/(\rho ck)_1$ and the term e on the curves describes the error δT arising in the measurement of the individual temperatures divided by the temperature difference $T_2^0 - T_1^0$. The error analysis itself is presented in the Appendix. As expected, the accuracy in the determination of the penetration property is greatest when the ratio $(\rho ck)_2/(\rho ck)_1$ is 1, but it decreases moderately for a range between 0.2 and 5 so that one material for the sampler can be used to measure penetration properties over a wide range. All typical construction materials with the exception of metals on one hand and insulation materials on the other hand have values within this range if a value $1.5 \times 10^6\ W^2 s/m^4 K^2$ is selected for sampler material.

Design analysis

Since it is impractical to use a very large instrument to simulate infinite solids, finite dimensions for the sampler must be chosen. The duration of the test must be limited so that the finite boundaries do not distort the temperature field in the central section. The sampler then can be considered as a semi-infinite solid and, under these conditions, equation (2) can be applied along the central axis of the sampler to predict the value of the penetration parameter (ρck) for the material being measured.

A cylindrical geometry is chosen for ease of manufacture and to reduce the analysis to two dimensions. For circular symmetry, the two dimensional heat-conduction equation is given by

$$k_i \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = \rho_i c_i \frac{\partial T}{\partial t}. \qquad (3)$$

Where the subscript i takes on the values 1, 2 and 3 referring to the respective solids following the notation

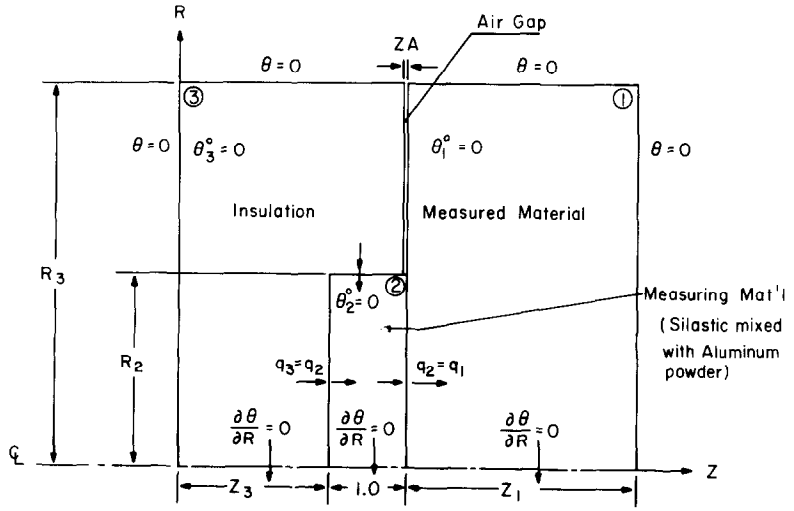

FIG. 5. Calculation domain of sampler and measured material with boundary conditions.

of Fig. 5. By introducing the following dimensionless variables

$$\theta = \frac{T - T_1^0}{T_2^0 - T_1^0},\ R = r/L,\ Z = z/L,\ Fo_i = \frac{\alpha_i t}{L^2} \quad (4)$$

where Fo_i is the Fourier number, a dimensionless parameter for the time scale, and α is the thermal diffusivity; the above equations transform to:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right) + \frac{\partial^2\theta}{\partial Z^2} = \frac{\partial\theta}{\partial Fo_i}. \quad (5)$$

The equations can be written in the following form for the materials 1, 2 and 3 respectively

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{R}\right) + \frac{\partial^2\theta}{\partial Z^2} = \frac{\partial\theta}{\partial Fo_1} \quad (6)$$

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right) + \frac{\partial^2\theta}{\partial Z^2} = \frac{\alpha_1}{\alpha_2}\frac{\partial\theta}{Fo_1} \quad (7)$$

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right) + \frac{\partial^2\theta}{\partial Z^2} = \frac{\alpha_1}{\alpha_3}\frac{\partial\theta}{\partial Fo_1}. \quad (8)$$

The calculation domain as well as the boundary and initial conditions are shown in Fig. 5. The initial uniform temperature of the insulation 3 is listed as equal to the initial temperature of the measured material 1. In fact, this assumption represents an extreme situation and minimizes the time available for the experiment. The heat flux has to be continuous on the boundaries between the media 1, 2 and 3. This requires

$$k_1\frac{\partial T}{\partial n_1} = k_2\frac{\partial T}{\partial n_2},\ k_2\frac{\partial T}{\partial n_2} = k_3\frac{\partial T}{\partial n_3} \quad (9)$$

with n indicating the normals to the boundaries or in dimensionless form

$$\left(\frac{\partial\theta}{\partial N}\right)_1 = \frac{\alpha_2}{\alpha_1}\left(\frac{\rho_2 c_2}{\rho_1 c_1}\right)\left(\frac{\partial\theta}{\partial N}\right)_2$$

$$\frac{\alpha_2}{\alpha_1}\cdot\left(\frac{\rho_2 c_2}{\rho_1 c_1}\right)\left(\frac{\partial\theta}{N}\right)_2 = \frac{\alpha_3}{\alpha_1}\left(\frac{\rho_3 c_3}{\rho_1 c_1}\right)\left(\frac{\partial\theta}{\partial N}\right)_3 \quad (10)$$

with $N = n/L$.

The present problem, therefore, depends on a number of dimensionless parameters: four physical (α_2/α_1), (α_3/α_1), $(\rho_2 c_2/\rho_1 c_1)$, and $(\rho_3 c_3/\rho_1 c_1)$ and five geometric, z_1, z_3, R_2, R_3 and ZA.

EXPERIMENTS

An instrument of the type described in the preceding pages was constructed to survey the thermal penetration property of concrete walls and floors in an underground building [6]. The following sections describe the instrument and measurements of the value ρck for a few materials to check the accuracy which can be obtained.

Selection of the sampler material

The sampler material should be reasonably elastic to ensure intimate contact with the concrete surface when pressed onto it by hand. Materials considered for this purpose include teflon as well as high and low density polyethylene. The values of the thermal penetration parameter of these materials are in the right range; however, these materials had to be discarded because the first two are not deformable under low hand-held pressure and the last material deforms permanently at temperatures above 40–50°C. The final choice was RTV-734, a Dow Corning silastic rubber which deforms easily, having a Durometer hardness of 35 Shore A, and remains elastic over the temperature range from −65 to 260°C. Its properties are listed together with the properties of concrete in Table 1. Observe that the ρck value for RTV-734 is somewhat low. It was, therefore, increased by mixing the RTV with aluminum powder; an amount of 10% by weight of aluminum powder, 325 mesh (0.01 mm size) manufactured by Alfa Products results in the properties included in Table 1. Increasing the aluminum content above 10% further increases the thermal penetration parameter but also increases the hardness of the material to an unacceptable degree.

Table 1. Material properties

Material	k [W/m K]	c [J/kg K]	ρ [kg/m³]	ρck [W²/s m⁴ K²]	$[\rho ck/(\rho ck)_c]^{1/2}$	γ [m²/s]
Concrete	1.00	880	2100	1.85×10^6	1	5.41×10^{-7}
Polyethylene (low density)	0.531	2300	910	1.11×10^6	0.775	2.54×10^{-7}
RTV-734	0.223	1870	1009	4.22×10^5	0.478	1.18×10^{-7}
RTV-734, 10% alum.	0.792	1670	1057	1.40×10^6	0.869	4.48×10^{-7}

The thickness of the sampler is limited since the liquid silastic 734 cures to a solid through a reaction with moisture which must diffuse through it. The maximum thickness which will cure in a reasonable time is 12 mm. A 104 mm diameter mold 24 mm thick was therefore used to cast the composite mixture. After the top has cured the mold is inverted and removed to allow the bottom to cure fully. The sampler material is then machined flat to the final thickness of 20 mm and 100 mm diameter.

Equipment and test procedure

The holder surrounding the sampler in Fig. 3 was manufactured of pinewood, which is strong mechani-

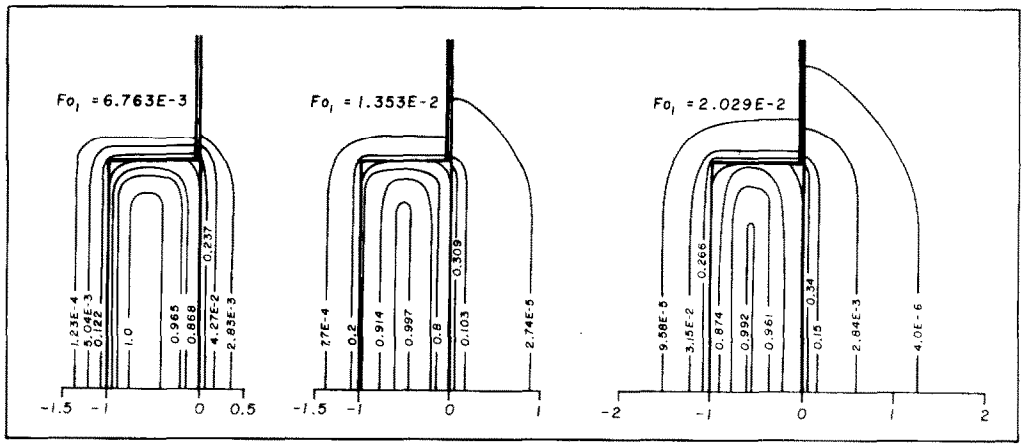

FIG. 6. Isothermal lines (θ = constant) in the sampler and material 1 for three different times.

cally. Its thermal conductivity has a value 0.17 W/mK which is sufficiently low to provide insulation to the sampler.

The hot plate in Fig. 3 which is used to bring the sampler to an elevated uniform temperature has an electric resistance heater of 50 W; another 30 W heater, located between the sampler and its holder, is used simultaneously to reduce the time required to bring the sampler to the desired elevated uniform temperature. Both heaters were grounded carefully to insure reliable thermocouple readings.

Three copper–constantan thermocouples are used to monitor the temperature on both sides and in the center of the sampler material. All of them were originally made of 30 AWG wires but after some initial tests the thermocouple which provides the reading of the interface temperature between the sampler and the concrete was changed to 50 AWG gage and was inserted into a slot in the silastic rubber as close as possible to the sampler surface. This was done to improve the accuracy of the measurement of the interfacial temperature and provide protection to the fine thermocouple.

The sampler is placed on the hot plate to start a test and both heaters are turned on. The power to each is adjusted continuously so as to keep the two outer thermocouple readings as nearly equal as possible. The temperature in the center of the sampler lags the outer temperature by a constant amount ΔT_H after an initial transient period. When the outer temperatures have exceeded the selected temperature by ΔT_H, the power to the two heaters is lowered so as to just make up thermal losses and is held at this value sufficiently long to allow the center temperature to approach the outer temperature closely. Once the three thermocouple readings agree to within 0.3°C the average of their readings is interpreted as the temperature T_1^0. The sampler is then removed from the hot plate and pressed against the material for which the thermal penetration property is to be determined. The original uniform temperature T_2^0 of the material under investigation was measured before it was covered by the sampler. The readings of the interface thermocouple at a period during which it does not change with time provides the temperature T_m required for the evaluation of equation (1).

RESULTS AND DISCUSSION

Computer simulation

The equations (6)–(8) with their initial and boundary conditions presented in the preceding section were solved numerically on a CDC 6600 computer.

The values of the dimensionless parameters involved were chosen so that the result of the computation is useful for the instrument to measure the thermal penetration property of concrete walls. They are:

$R_2 = 2.5, R_3 = 5, Z_1 = 3, Z_2 = 1, Z_3 = 2, ZA = 0.04$

$\alpha_2/\alpha_1 = 0.828$ (RTV − 734 + 10% Al/concrete)

$\alpha_3/\alpha_1 = 0.213$ (pinewood/concrete).

The approximate values $\rho_2 c_2/\rho_1 c_1 \simeq \rho_3 c_3/\rho_1 c_1 \simeq 1$ were used for the ratios of specific heats per unit volume.

The results of the numerical solution at three different times were used to draw isothermal lines shown in Fig. 6. Observe that the side boundaries are safely located so that the assumptions shown in Fig. 5 hold even for the longest time period $Fo_1 = 2.029 \times 10^{-2}$. Note the shrinkage of the hot domain $\theta = 1$ within the sampler material 2. For $Fo_2 = 2.029 \times 10^{-2}$, this domain has just disappeared leaving the isotherm 0.992. This is considered acceptable and gives the maximum time for the measurement for the above values of the dimensionless parameters. It also may be observed that during that time rim effects do not influence the temperature field at the axis of the sampler where the temperatures are measured.

Figure 7 shows the variation of the temperature of the interface between the sampler material and the measured material 1 with time. The fluctuation of this dimensionless temperature θ_m for the Fourier modulus close to zero is due to the finite difference approximation.

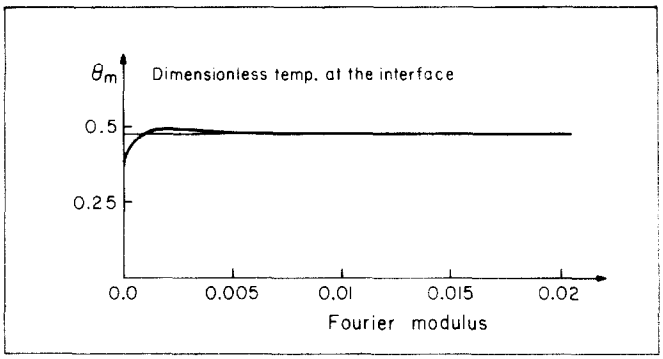

FIG. 7. Temperature variation of the interface between sampler material and measured material 1 with time.

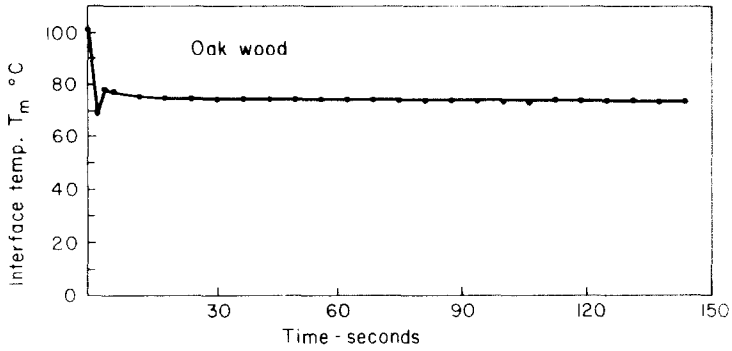

FIG. 8. Interface temperature T_m with time for oakwood.

Table 2. Comparison of measured results with available data

Material	T_1 [°C]	T_2 [°C]	T_m [°C]	$(\rho ck)_{meas.}$ [W^2s/m^4K^2]	$(\rho ck)_{publ.}$ [W^2s/m^4K^2]	Difference [%]
Plexiglass	24.25	102.9	72.0	5.85×10^5	5.78×10^5	1.4
Teflon	24.2	105.8	69.5	8.99×10^5	8.95×10^5	0.5
Oak wood, radial	20.18	101.5	74.8	3.33×10^5	3.17×10^5	5.3

Experiments

Results were obtained for three different materials to check the accuracy of this method. Table 2 lists the results and the comparison with available data.

The value of the product $(\rho ck)_{publ.}$ is obtained from published information by Cadillac Plastic [6], the manufacturer of the Plexiglass and Teflon used in the experiment. The values for oakwood listed in Table 2 are from ASHRAE [7], while Eckert and Drake [5] give a range for radial oakwood from 2.47×10^5 to 4.02×10^5 W^2s/m^4K^2. This suggests that transfer properties of wood are influenced by the moisture content or by the species of oak. It is worth noting that a test of the oakwood sample before drying resulted in the value $\rho ck = 6.25 \times 10^5$ W^2s/m^4K^2. Figure 8 presents the interface temperature T_m as a function of time for oakwood. Semi-infinite behavior continues to exist even after the center temperature has dropped below the initial temperature of silastic by something like 0.5°C. The drop in the first part of the temperature–time curve below the final values, shown in Fig. 8 is due to cooling of the location of the thermocouple in the sampler by ambient air during the time required to move the sampler from the hot plate to the specimen. The overshoot of the temperature after the initial dropoff is caused by a heat flux to this location from the surrounding sampler material.

CONCLUSION

The thermal penetration property sampler is an accurate instrument that can be used quickly and conveniently to measure *in-situ* the value of the product (ρck). It can also be used to study the change in moisture content of various materials. It should be useful for measurements and monitoring of the thermal characteristics of structures in buildings.

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APPENDIX

With the abbreviation

$$a = \left[\frac{(\rho ck)_2}{(\rho ck)_1}\right]^{1/2} \tag{A1}$$

equation (2) assumes the form

$$a^2 = \frac{T_m - T_1^0}{T_2^0 - T_m}. \tag{A2}$$

With the dimensionless temperature

$$\theta_m = \frac{T_m - T_1^0}{T_2^0 - T_1^0} \tag{A3}$$

it becomes

$$a = \left(\frac{\theta_m}{1-\theta_m}\right)^{1/2}. \tag{A4}$$

Now we assume that an error δT, or in dimensionless form

$$e = \frac{\delta T}{T_2^0 - T_1^0} \tag{A5}$$

occurs in the measurement of each of the three required temperatures. This will possibly cause a maximum error in the calculation of the parameter a which assumes the value described by the equation

$$a_{\text{err}}^2 = \frac{T_m - T_1 + 2\delta T}{T_2 - T_m - 2\delta T} = \frac{\theta_m + 2e}{1-(\theta_m + 2e)}. \tag{A6}$$

The relative error in percent in the determination of the ratio of the thermal penetration properties therefore is given by

$$E = \frac{1}{a^2}\left[\frac{\theta_m + 2e}{1-(\theta_m+2e)} - a^2\right] \times 100. \tag{A7}$$

Figure 4 shows this error as a function of the ratio of the thermal penetration properties with the relative measurement error e as parameter.

UN INSTRUMENT DE MESURE DE LA PROPRIETE THERMIQUE DE PENETRATION ρck (EFFUSIVITE)

Résumé—On décrit un instrument de mesure du produit de la masse volumique, de la chaleur massique et de la conductivité thermique des matériaux solides, paramètre qui intervient fréquemment dans les problèmes de conduction en régime variable. Les résultats sont donnés avec la précision qui peut être obtenue.

EIN INSTRUMENT ZUR MESSUNG DES THERMISCHEN EINDRINGENS VON $\rho ck(\rho c\lambda)$

Zusammenfassung—Es wird ein Instrument beschrieben, welches das Produkt der Dichte, der spezifischen Wärmekapazität und der Wärmeleitfähigkeit von festen Körpern mißt, ein Parameter, der oft in der Analyse von instationären Wärmeleitprozessen auftritt. Die Ergebnisse der Messungen werden beschrieben und zeigen die Genauigkeit, die erreicht werden kann.

ПРИБОР ДЛЯ ИЗМЕРЕНИЯ ТЕПЛОВОЙ АКТИВНОСТИ

Аннотация — Описывается прибор, измеряющий произведение плотности, удельной теплоемкости и теплопроводности твердых материалов, т. е. параметр, который часто встречается при анализе нестационарных процессов теплопроводности. Приводятся результаты измерений, демонстрирующие диапазон точности прибора.